ANALYSIS OF SOME PARAMETERS OF A LIGHT FLASH, ACCOMPANYING THE COLLAPSE OF A CAVITY, INITIATED BY AN ELECTRICAL DISCHARGE IN WATER

P. I. Golubnichii, V. M. Gromenko,
and A. D. Filonenko

UDC 534.29:535.37

In [1, 2], an attempt based on the experimental results and theoretical estimates is made to explain some characteristics of the form of the light pulse emitted by a vapor-gas cavity at the last stage of its collapse. In particular, in some cases the so-called recombination mechanism of emission, which is related to the phenomenon of "quenching" of the plasma forming during the electrical discharge or some other powerful form of energy liberation in the liquid may dominate.

Keeping to this point of view of the nature of the light flash (realized primarily for not very large ratios of the maximum to minimum radii, i.e., $r_0/r \sim 10$-20), it is possible to find a number of regularities from a simultaneous analysis of the equations of motion of the walls of the spherical cavity and the equations of kinetics of recombination of electrons and ions.

The first integral of Rayleigh's equation for a spherical bubble filled with gas [3]

$$v^2=\frac{2}{3}\frac{p_{g0}}{\rho}\frac{1}{\gamma-1}\left[\left(\frac{r_0}{r}\right)^3-\left(\frac{r_0}{r}\right)^{3\gamma}\right]-\frac{2}{3}\frac{p_0}{\rho}\left[1-\left(\frac{r_0}{r}\right)^3\right], \quad (1)$$

where p_{g0} is the pressure of the gas for a bubble radius of r_0, ρ is the density of the liquid, γ is the index of the adiabat, and p_0 is the atmospheric pressure, can be put into the form

$$v=\pm\frac{3}{2}x^{3/2}\frac{r_0}{\tau}\left(\frac{x_0-x}{x_0-1}\right)^{1/2}, \quad (2)$$

if the well-known expression [3] for the period of collapse τ and the ratio $x_0 = r_0/r_m$ of the maximum to minimum value of the bubble radius for $\gamma = 4/3$ and $x_0 \gg 1$ are used. Estimates show that the function v, where the variable $x = r_0/r$, reproduces (1) with quite high accuracy and that at x = 1 it does not differ significantly from zero. On the whole the difference between (1) and (2) lies in neglecting the 1 compared to the term x^3, which in the region $x \sim x_0$ is three to four orders of magnitude greater.

Supplementing the equation for kinetics of recombination [4] with the term $-3n(\dot{r}/r)$, which takes into account the increase in the density of charged particles during the collapse of the sphere, we obtain the expression

$$dn/dt=-An^3T^{-9/2}-3n(\dot{r}/r), \quad (3)$$

where n is the density; T is the temperature, which depends on the radius for an adiabatic process ($\gamma = 4/3$) as $T = T_0r_0/r$; A is a constant; and r is the instantaneous radius.

After making the substitution T = T(r) and replacing r with x, using the expression for $\dot{r}$ (2), from (3) we obtain the equation

$$dn+[Bx^{-8}(x_0-x)^{-1/2}n^3-3nx^{-1}]dx=0, \quad (4)$$

which will be the equation in terms of total differentials with the integration factor x^6/n^3 (B is a constant). The solution of this equation is the function

$$B\left[\frac{(x_0-x)^{1/2}}{x_0x}+\frac{1}{2x_0^{3/2}}\ln\frac{x_0^{1/2}+(x_0-x)^{1/2}}{x_0^{1/2}-(x_0-x)^{1/2}}\right]+\frac{1}{2}n^{-2}x^6=C, \quad (5)$$

Voroshilovgrad. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 1, pp. 3-6, January-February, 1984. Original article submitted November 30, 1982.

0021-8944/84/2501-0001$08.50

where the constant C can be determined from the initial condition, i.e., $x = 1$, $n = n_1$ or, in other words, the value of C can be found knowing the density n_1 of charged particles at the time corresponding to the maximum radius of the cavity.

It is well known that the amplitude of the light flash A_ν, due to the photorecombination with binary collisions [4], is proportional to $n^2 T^{-3/4} r^3$ and substituting into (5) has the form

$$A_\nu \sim \frac{x^{9/4}}{n_1^{-2} + 2B\,[f(1) - f(x)]}, \tag{6}$$

where f(x) is the function enclosed in brackets in Eq. (5).

To analyze (6) further, it is necessary to note that the quantity n_1^{-2} is much less than the second term and can be neglected. This conclusion is based on experimental measurements of the absolute light flux at the stage of maximum expansion of the cavity, when the particle density is very low and photorecombination proceeds only by means of binary collisions.

Examining the behavior of the function $A_\nu(x)$ in the vicinity of the plate $x = x_0$, we arrived at the conclusion that the form of the light pulse must be asymmetric, which was observed experimentally (see Fig. 2 in [2]). This property of the function $A_\nu(x)$ follows from the fact that the sign of the second term in brackets in the denominator changes after the variable x reaches its maximum value x_0 [in this case the negative sign of $(x_0 - x)^{1/2}$ must be used] and further decrease of x (which is equivalent to increasing the diameter of the cavity) leads to a steeper change of the function A_ν.

The following characteristic of the light pulse follows from the property that the first derivative of $A_\nu(x)$ with respect to x is negative at the point $x = x_0$, i.e., the extremum of this function is located to the left of the point x_0. This means that the maximum of the emission intensity does not coincide with the moment of maximum compression of the cavity. This effect is apparently characteristic only for the recombination mechanism of luminescence (in contrast, for example, to the thermal mechanism).

To check the leading effect experimentally, it is necessary to estimate Δt in terms of experimentally determined parameters. Equating the derivative of $A_\nu(x)$ to zero and using the properties of the function f(x) in the vicinity of x_0, we find

$$[(x_0 - x)/x_0]^{1/2} x = 4/9. \tag{7}$$

To determine the explicit dependence of the quantity $(x_0 - x)^{1/2}$ on time, we integrate (2):

$$t = \int_{r_0}^{r} \frac{dr}{v} = \tau \left\{1 - \frac{4}{3}\frac{(x_0 - 1)^{1/2}}{x_0^3}\left[\frac{1}{5}\left(\frac{x_0}{x} - 1\right)^{5/2} + \frac{2}{3}\left(\frac{x_0}{x} - 1\right)^{3/2} + \left(\frac{x_0}{x} - 1\right)^{1/2}\right]\right\}. \tag{8}$$

Here τ is the collapse time; t is the time interval during which the radius of the cavity changes from r_0 to r (or, correspondingly, from $x = 1$ to x). In the region $x \gtrsim x_0$ only the last term makes a significant contribution during this interval, so that taking into account (7) and (8) we obtain

$$\Delta t = (16/27)\,\tau x_0^{-7/2}, \tag{9}$$

where Δt is the time interval between the moment of maximum emission and the moment at which the cavity reaches its minimum size.

It is easiest to determine the half-width of the light-signal relation between the form of the pulse and the parameters of the cavity. To obtain this dependence, the interval (9), which according to the experimental data is much less than the duration of the pulse itself, can be neglected and it may be assumed that the maximum of $A_\nu(x)$ coincides with the point x_0. In this case from the system of equations

$$\frac{x_1^{9/4}}{f(1) - f(x_1)} = \frac{x_0^{9/4}}{2f(1)}, \quad \frac{x_2^{9/4}}{f(1) + f(x_2)} = \frac{x_0^{9/4}}{2f(1)},$$

where x_1 and x_2 are points at which the quantity $A_\nu(x)$ equals one half the amplitude, we find

$$[(x_0 - x)/x]^{1/2} = 0.6\left(1 \pm x_0^{-1}\right),$$

and, in addition, the plus sign corresponds to the value of $x = x_1$ and the minus sign corresponds to x_2. Here, as above, we use the first-order terms in the expansion. The intervals obtained can be expressed as explicit functions of time with the help of (8), after which the half-width of the light pulse σ_+, which represents the sum of two segments of time at half-height, will be determined in the form

$$\sigma_+ = 2\tau x_0^{-5/2} \qquad (10)$$

and the difference between the segments σ_- can be characterized by the degree of asymmetry of the pulse

$$\sigma_- = 3\tau x_0^{-7/2}, \qquad (11)$$

i.e., it will reflect to some extent the presence of the recombination character of the emission.

Formula (10) was checked experimentally for some liquids ($CHCl_3$, CCl_4, H_2O) with different values of x_0. The validity of relation (10) was, on the whole, confirmed, although significant disagreements were observed in some cases. It should be noted that the main experimental problem was to monitor reliably the form of the surface of the smallest bubbles. Certain success was achieved using a recording apparatus with an electro-optical converter (LV-05) and, in this case, in order to eliminate the noise created by the photomultiplier (used to record the shape of the pulse), the light source (pulsed lamp) and the photomultiplier (FÉU) operated in different regions of the visible range.

The experimental data show that the form of the cavity at the last stages of collapse depends very strongly on the interelectrode distance and for steady-state power of the electrical discharge it is possible to select a gap for which the form of the cavity is virtually spherical.

As far as checking Eq. (11) is concerned (for bubbles with maximum radius $\sim$1 cm), it should be noted that the use of the shadow method (see, for example, [1]) introduces an error related to the determination of the time at which the minimum radius is attained, which does not permit giving a definite answer.

In conclusion, it should be noted that the dependence obtained for the half-width of the pulse is interesting primarily because knowing this parameter (which is easily determined experimentally, as also τ), it is possible to find the quantity $x_0 = r_0/r_m$, which is very important for analyzing different processes and which is quite difficult to measure for large cavities and, apparently, entirely impossible to measure at the present time for microbubbles (for example, in ultrasonic cavitation).

LITERATURE CITED

1. P. I. Golubnichii, V. M. Gromenko, and A. D. Filonenko, "On the nature of the electrohydrodynamic sonoluminescence pulse," Zh. Tekh. Fiz., 50, No. 11 (1980).
2. P. I. Golubnichii, V. M. Gromenko, V. G. Kudlenko, and A. D. Filonenko, "On electromagnetic radiation accompanying the collapse of a vapor-gas cavity formed by an intense liberation of energy in a liquid," in: Nonstationary Problems of Hydrodynamics [in Russian], No. 48, Institute of Geology, Siberian Branch, Academy of Sciences of the USSR, Novosibirsk (1980).
3. A. D. Pernik, Problems of Cavitation [in Russian], Sudostroenie, Leningrad (1966).
4. Ya. B. Zel'dovich and Yu. P. Raizer, Physics of Shock Waves and High-Temperature Hydrodynamic Phenomena [in Russian], Nauka, Moscow (1966).